Physics

Key Skills to develop and refine	Using vectors. Algebra and logarithms. Dimensional analysis. Graphing. MS Excel.
1.	Watch through this playlist of videos, completing the tasks below after the appropriate videos. https://youtube.com/playlist?list=PLhWoFJbwn00wDnH6wvnm0TzbjErRqsyU_
2.	Answer the questions in this document. Check your answers where they are available.
Compulsory task	All of the above, and make a note of any difficulties you encountered to discuss when you start in September.
Autumn preparation	There will be a test at the start of Year 12 about this summer work. Be ready for it.

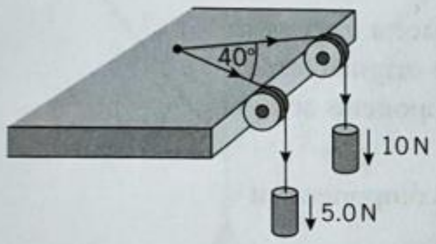
Bridging the Gap: GCSE to A Level Physics

Lesson 1: Vectors

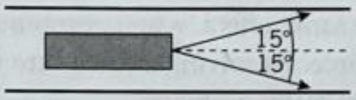
Low demand questions

2 For each of these situations draw a triangle or polygon of forces to determine the resultant force:

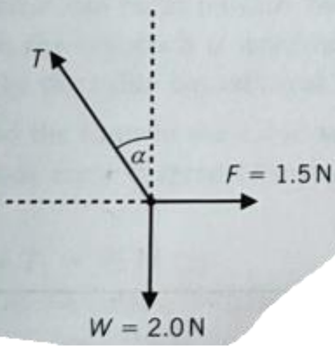
a



b Two forces of 5 kN towing a boat



3 These three forces are in equilibrium.
Draw a triangle of forces to find T and α.



4 Find the resultant force for these pairs of forces at right angles:

a 3.0 N and 4.0 N **b** 5.0 N and 12.0 N

You might have to look up 'normal reaction force' again for this, but it is GCSE Level

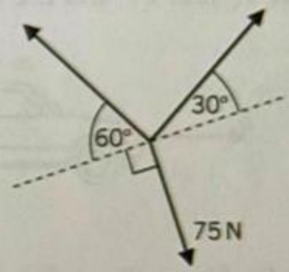
1 A force of 550 N is applied to a box at an angle of 30° to the horizontal. Calculate the horizontal and vertical components of the force.

2 Calculate the normal reaction and the friction for a box of weight 85 N in equilibrium on a slope of angle 15°.

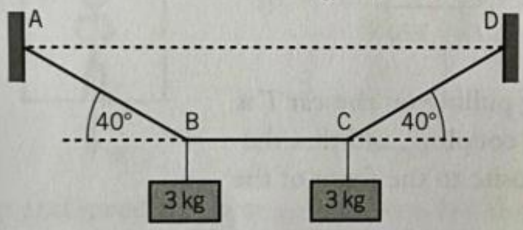
Medium demand questions

PRACTICE QUESTIONS

3 The three strings in the diagram are in tension and in equilibrium. Calculate the tension in each string.



4 Two masses are supported by three strings. BC is horizontal. What is the tension in string AB, BC, and CD?



5 A cable, parallel to a slope 30° to the horizontal, pulls a block up the slope at a steady speed. The block weighs 65 N and the friction with the slope is 12 N. What is the tension in the cable, and the normal reaction force?

High demand questions

42. (I) Draw the vector $3\mathbf{i} + 4\mathbf{j}$ by first drawing the x-component vector, then the y-component vector, then adding them graphically. Multiply the vector by a factor of two and repeat the exercise.

43. (I) A drunken sailor stumbles 4 paces north, 6 paces northeast, 2 paces east, and 5 paces west. Describe the final location from the initial position by a single displacement vector.

44. (I) What is the resultant vector when the vectors $\mathbf{A} = 6\mathbf{i} - 5\mathbf{j}$ and $\mathbf{B} = 8\mathbf{i} + 3\mathbf{j}$ are added together? When $\mathbf{B}$ is subtracted from $\mathbf{A}$?

45. (II) A football player catches the kickoff on the 5-yd line and runs straight up the field for 20 yd, turns left for 15 yd, goes straight up the field for 10 yd, turns right for 25 yd, reverses his field (makes a 180° turn) for 10 yd, and then streaks straight up the field for a touchdown. Define a coordinate system and list the entire path in vector form.

46. (II) Draw a vector $\mathbf{V}$ that points in the northwesterly direction, making an angle α with the northerly direction, as in Fig. 1–27. If north is chosen as the $+y$-direction and east as the $+x$-direction, what is the x-component of $\mathbf{V}$?

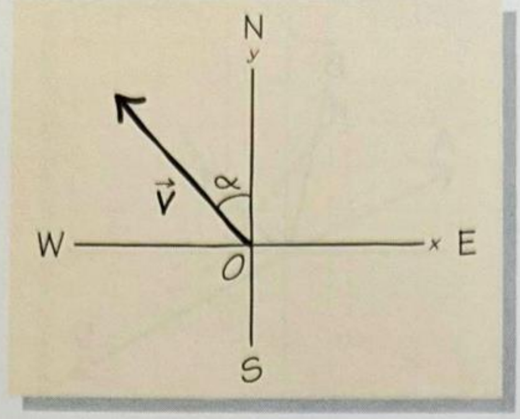


FIGURE 1–27 Problem 46.

47. (II) Suppose that in Problem 46 you choose north as the $+x$-direction and west as the $+y$-direction. What is the x-component of $\mathbf{V}$ in this case?

48. (II) Refer to the situation outlined in Problems 46 and 47. Choose the $+x$-axis as the line that makes an angle of 45° with the northerly direction and is inclined to the east, and the $+y$-axis as the line that makes a 45° angle with the westerly direction and is inclined to the north. What is the x-component of $\mathbf{V}$ in this case?

50. (II) Consider the following vectors: $\mathbf{A} = -2\mathbf{i} - 3\mathbf{j}$; $\mathbf{B} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$; $\mathbf{C} = 3\mathbf{j} + 3\mathbf{k}$; and $\mathbf{D} = -2\mathbf{i} - \mathbf{k}$. Find (a) $\mathbf{A} + \mathbf{B} + \mathbf{C} + \mathbf{D}$; (b) $\mathbf{A} - \mathbf{D}$; (c) $\mathbf{A} + \mathbf{D} - \mathbf{B}$; and (d) $|\mathbf{A} - \mathbf{C}|$.

51. (II) Vectors **A**, **B**, **C**, and **D** are shown in Fig. 1–29. (a) Give the vectors in component form. (b) Determine the following quantities both algebraically and graphically: $2\mathbf{A} + \mathbf{C} - \mathbf{D}$, $\mathbf{B} + \mathbf{C}/2$, $|\mathbf{D} - \mathbf{B}|$.

52. (II) Suppose that you have three vectors, $\mathbf{A} = 3\mathbf{i} + 4\mathbf{j}$, $\mathbf{B} = 2\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$, and $\mathbf{C} = -\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}$. Show that the sum of these three vectors can alternatively be computed by first summing **A** and **B** and then summing the resultant with **C**, or by first summing **B** and **C** and then summing the resultant with **A**.

Summary questions

1 a Convert the following angles from degrees into radians and express your answer to one further significant figure than in each question:

i $30°$,

ii $50°$,

iii $120°$,

iv $230°$,

v $300°$.

b Convert the following angles from radians into degrees and express your answer to one further significant figure than in each question:

i 0.10 rad,

ii 0.50 rad,

iii 1.20 rad,

iv 2.50 rad,

v 6.00 rad.

2 a Measure the diameter of a 1p coin to the nearest millimetre. Calculate the angle subtended at your eye, in degrees, by a 1p coin held at a distance of 50 cm from your eye.

b i Estimate the angular width of the Moon, in degrees, at your eye by holding a millimetre scale at 50 cm from your eye and measuring the distance on the scale covered by the lunar disc.

ii The diameter of the Moon is 3500 km. The average distance to the Moon from the Earth is 380 000 km. Calculate the angular width of the Moon as seen from the Earth and compare the calculated value with your estimate in **b i**.

3 a Use the small angle approximation to calculate $\sin\theta$ for $\theta =$

i $2.0°$,

ii $8.0°$.

b Show that the small angle approximation for $\sin\theta$ is more than 99% accurate for $\theta = 10°$.

4 Use your calculator to find

i $\sin\theta$,

ii $\cos\theta$ for the following values of θ:

a 0.1 rad,

b $10°$,

c $45°$,

d 0.25π rad.

Answers

1 a i 0.524 rad
ii 0.873 rad
iii 2.094 rad
iv 4.014 rad
v 5.236 rad
b i 5.73°
ii 28.7°
iii 68.8°
iv 143.2°
v 343.8°

2 a 20 mm, 2.3°
b ii 0.5°

3 a i 0.035
ii 0.140

4 a i 0.0998
ii 0.995
b i 0.1736
ii 0.9848
c i 0.7071
ii 0.7071
d i 0.7071
ii 0.7071

Summary questions

1 Solve each of the following pairs of simultaneous equations.

a $3x + y = 6; 2y = 5x + 1$

b $3a - 2b = 8; a + b = 2$

c $5p + 2q = 18; q = 2p$

2 Use the data and the given equation to write down a pair of simultaneous equations and so determine the unknown quantities in each case:

a For $v = u + at$, when $t = 3.0$ s, $v = 8.0$ m s^{-1} and when $t = 6.0$ s, $v = 2.0$ m s^{-1}. Determine the values of u and a.

b For $\varepsilon = IR + Ir$, when $R = 5.0\ \Omega$, $I = 1.5$ A and when $R = 9.0\ \Omega$, $I = 0.9$ A. Determine the values of ε and r.

3 Solve each of the following quadratic equations.

a $2x^2 + 5x - 3 = 0$

b $x^2 - 7x + 8 = 0$

c $3x^2 + 2x - 5 = 0$

4 Use the data and the given equation to write down a quadratic equation and so determine the unknown quantity in each case:

a $s = ut + \frac{1}{2}at^2$, where $s = 20$ m, $u = 4$ m s^{-1} and $a = 6$ m s^{-2}; find t.

b $P = V^2 \dfrac{R}{(R+r)^2}$, where $P = 16$ W, $V = 12$ V, $r = 2.0\ \Omega$; find R.

Answers

1 a $x = 1, y = 3$

b $a = 2.4, b = -0.4$

c $p = 2, q = 4$

2 a $u = 14$ m s^{-1}, $a = -2.0$ m s^{-2}

b $r = 1.0\ \Omega$, $\varepsilon = 9.0$ V

3 a 0.5 or −3

b 1.4 or 5.6

c $-\dfrac{10}{6} = -1.67$ (to 3 sig. figs) or 1

4 a $t = -\dfrac{20}{6} = -3.33$ (to 3 sig. figs) or 2 s

b $R = 1$ or $4\ \Omega$

Summary questions

1 a Use your calculator to work out

i $\log_{10} 3$

ii $\log_{10} 15$

b Use your answers in **a** to work out

i $\log_{10} 45$

ii $\log_{10} 5$

2 The gain of an amplifier, in decibels, is given by the formula $10 \log_{10} \frac{V_{out}}{V_{in}}$.

a Calculate the gain, in decibels (dB), for

i $V_{out} = 12V_{in}$

ii $V_{out} = 5V_{in}$

b Show that the gain, in decibels, of an amplifier for which $V_{out} = 60V_{in}$ is equal to the sum of the gain in **a i** and the gain in **a ii** above.

3 Write down the gradient and the y-intercept of a line on a graph representing the equation $\log_{10} y = n \log_{10} x + \log_{10} k$ for

a $y = 3x^5$

b $y = \frac{1}{2}x^3$

c $y = x^2$

4 a Use your calculator to work out

i $\ln 3$

ii $\ln 15$

b Use your answers in **a** to work out

i $\ln 45$

ii $\ln 5$

Answers

1 a i 0.477

ii 1.176

b i 1.653

ii 0.699

2 a i 10.8 dB

ii 7.0 dB

b 17.8 dB

3 a $n = 5, k = 3$

b $n = 3, k = \frac{1}{2}$

c $n = 2, k = 1$

4 a i 1.10

ii 2.71

b i 3.81

ii 1.61

Summary questions

1 a For each exponential decrease equation, write down the initial value at $t = 0$ and the decay constant:

i $x = 2e^{-3t}$

ii $x = 12e^{-t/5}$

iii $x = 4e^{-0.02t}$

b For each exponential decrease equation above, work out the half–life.

2 A radioactive isotope has a half-life of 720 s and it decays to form a stable product. A sample of the isotope is prepared with an initial activity of 12.0 kBq. Calculate the activity of the sample after:

a 1 min,

b 5 min,

c 1 h.

3 A capacitor of capacitance 22 μF discharged from a pd of 12.0 V through a 100 kΩ resistor.

a Calculate:

i the time constant of the discharge circuit,

ii the half-life of the exponential decrease.

b Calculate the capacitor pd

i 2.0 s, and

ii 5.0 s after the discharge started.

4 A certain exponential decrease process is represented by the equation

$x = 1000e^{-5t}$

a i Calculate the half-life of the process.

ii Calculate x when $t = 0.5$ s.

b Show that the above equation can be rearranged as an equation of the form $\ln x = a + bt$ and determine the values of a and b.

Answers

1 a i 2, 3

ii 12, 0.2

iii 4, 0.02

b i 0.23 s

ii 3.5 s

iii 35 s

2 a 11.3 kBq

b 9.0 kBq

c 0.38 kBq

3 a i 2.2 s

ii 1.52 s

b i 4.83 V

ii 1.24 V

4 a i 0.14 s

ii 82

b $a = 6.9$, $b = -5$

1-4 DIMENSIONAL ANALYSIS

Dimensions

Experience has shown that there are three basic ways to describe any physical quantity: the space it takes up, the matter it contains, and how long it persists. All descriptions of matter, relationships, and events are combinations of these three basic characteristics. All measurements can be reduced ultimately to the measurement of length, time, and mass. Any physical quantity, no matter how complex, can be expressed as an algebraic combination of these three basic quantities. Speed, for example, is a length per time.

Length, time, and mass therefore have a significance far beyond that of providing the basis of a system of units. They specify the three **primary dimensions**. We use the abbreviations $[L]$, $[T]$, and $[M]$ for these primary dimensions. The **dimension** of a physical quantity is the algebraic combination of $[L]$, $[T]$, and $[M]$ from which the quantity is formed. The speed v provides an example. The dimension of v is

$$[v] = [L/T] \quad \text{or} \quad [LT^{-1}].$$

Do not confuse the dimension of a quantity with the units in which it is measured. A speed may have units of meters per second, miles per hour, or, for that matter, light-years per century. All of these different choices of units are consistent with the dimension $[LT^{-1}]$. In what follows, the square brackets, as used here, indicate that we are dealing with dimensions.

Any physical quantity has dimensions that are algebraic combinations $[L^qT^rM^s]$ of the primary dimensions, where the superscripts q, r, and s refer to the order (or power) of the dimension. Thus, for example, an area has dimension $[L^2]$. If all of the exponents q, r, and s are zero, the combination will be dimensionless. The exponents q, r, and s can be positive integers, negative integers, or even fractional powers.

Dimensional Analysis

Study of the dimensions of an equation—*dimensional analysis*—is an important exercise with several different uses in physics. Any equation that relates physical quantities must have consistent dimensions; that is, *the dimensions on one side of an equation must be the same as those on the other side.* This provides a valuable check for any calculation. Dimensional analysis can also reveal *scaling laws* (see Section 1–7), which describe how changing one quantity in a physical situation requires changes in others. Finally, when there is reason to believe that only certain physical quantities can enter into a physical situation, dimensional analysis can provide us with powerful insights.

Let's look at some examples of dimensional analysis. In Chapter 7, we derive a relation between the height h of a dropped object and the speed of that object. This relation involves the *acceleration of gravity*, g, a quantity whose dimension is $[g] = [LT^2]$. The relation reads

$$gh = \frac{1}{2}v^2.$$

Let's compare the dimensions on each side of this equation. The dimension of h is $[L]$, so the left-hand side has dimensions $[LT^{-2}][L] = [L^2T^{-2}]$. The right-hand side has the dimensions of speed squared, $[LT^{-1}]^2 = [L^2T^{-2}]$. Thus the dimensions match. If, through error, we had written a relation $gh^2 = \frac{1}{2}v^2$, then this check would have revealed the error. Note that dimensional analysis does not help us understand the numerical factor $\frac{1}{2}$.

EXAMPLE 1–2 Newton's law of universal gravitation gives the force between two objects of mass, m_1 and m_2, separated by a distance r, as

$$F = G\left(\frac{m_1m_2}{r^2}\right).$$

Use dimensional analysis to find the units of the gravitational constant, G.

Solution: First, the dimensions of the two sides of the equation must match. In the previous section, we learned that the unit of force is the newton, equivalent to $kg \cdot m/s^2$. Using these units, the dimensions of force must be $[MLT^{-2}]$. We now know the dimensions of every quantity in the equation for gravitational force except G. Writing the dimensions for both sides gives

$$[MLT^{-2}] = [G][M][M][L^2] = [G][M^2L^{-2}].$$

Note that the individual dimensions can be consolidated inside the square brackets or left within their own brackets—whichever is easiest. We solve for the dimension of G as

$$[G] = \frac{[MLT^{-2}]}{[M^2L^{-2}]} = [MLT^{-2}][M^{-2}L^2] = [M^{-1}L^3T^{-2}].$$

1–4 Dimensional Analysis

21. (I) The kinetic energy of a baseball is denoted by $mv^2/2 = p^2/2m$, where m is the baseball's mass and v is its speed. This relation can be used to define p, the baseball's momentum. Use dimensional analysis to find the dimensions of momentum.

22. (I) One of Einstein's most famous results is contained in the formula $E = mc^2$, where E is the energy content of the mass m, and c is the speed of light. What are the dimensions of E?

23. (I) A length L that appears in atomic physics is given by the formula $L = h/m_e c$, where m_e is the mass of an electron, c is the speed of light, and h is a constant known as Planck's constant. What are the dimensions of h?

24. (II) What are the dimensions of $h^2/m^3 G$, where h is a constant called Planck's constant, m is a mass, and G is the gravitational constant? The dimensions of the constants in this formula can be found in the list of physical constants given in Appendix II.

25. (III) A force F acting on a body of mass m a distance r from some origin has magnitude $F = Ame^{-\alpha r}/r^3$, where A and α are both constants. The constant $e = 2.718\ldots$ is Euler's constant. Given that the force has dimensions $\mathrm{kg \cdot m/s^2}$, what are the dimensions of (a) the constant α? (b) the constant A?

*1–7 The Uses of Dimensional Analysis

53. (II) We have seen in the text that the period of a simple pendulum is independent of the mass of the pendulum bob. Further, we have seen that the dimensional relation between the period, τ, the length, ℓ, of the pendulum, and the acceleration of gravity, g, takes the form

$$[\tau] = [\ell^r][g^s].$$

Use the fact that the dimension of τ is [T], that of ℓ is [L], and that of g is $[L/T^2]$ to show that

$$\tau = \sqrt{\frac{\ell}{g}}.$$

54. (II) In quantum mechanics, the fundamental constant called Planck's constant, h, has dimensions of $[ML^2T^{-1}]$. Construct a quantity with the dimensions of length from h, a mass m, and c, the speed of light.

55. (II) It is known that the quantity Ke^2/hc is dimensionless (K is a numerical constant; h and c are as discussed in Problem 54). (a) What are the dimensions of e? (b) What are the dimensions of e^2/R, where R is a length?

56. (II) You are told that the speed of sound in a metal depends only on the density ρ ($[ML^{-3}]$) and on the bulk modulus of the metal, B, which has dimensions $[ML^{-1}\, T^{-2}]$. Express the sound speed in terms of ρ and B.

72. (II) A mouse is 10 cm in length, whereas an elephant is 4 m in length. The amount of food an animal must eat is proportional to its heat loss, and the heat loss is proportional to its surface area. Compare the percentage of body weight that a mouse and an elephant must eat each day. Ignore the detailed differences in shape between an elephant and a mouse.

81. (III) A stretched wire has three physical attributes: the density λ, or mass per unit length; the total length ℓ; and the tension τ. The latter is related to how hard the wire is being pulled to keep it stretched, and has dimensions of $[MLT^{-2}]$. Show by dimensional analysis that if the time t_0 of one back-and-forth vibration of the wire in a direction perpendicular to its length depends only on these three quantities, then t_0 has the form $t_0 = (\text{a constant})\ \ell\sqrt{\lambda/\tau}$.

66. (II) The gasoline usage rate required to propel an automobile is very roughly proportional to the mass of the automobile. Assuming that the proportions and types of materials of an automobile do not change, calculate the percentage gasoline savings that would be realized if cars were reduced by 12 percent in all their dimensions.

67. In aquatic animals, the energy E available for motion is proportional to the mass of the animal, and the friction F with their skin is proportional to the surface area. All such animals have the same density, very close to that of water. If the maximum speed v such an animal can reach varies as $\sqrt{E/F}$, show that v is also proportional to $\sqrt{L}$, where L is some length characterizing the animal's size.

21. $[MLT^{-1}]$.
23. $[ML^2T^{-1}]$.
25. (a) $[\alpha] = [L^{-1}]$, (b) $[A] = [L^4T^{-2}]$.

Graphing skills questions

Question 1

Equation: $a = \delta\Omega + A$

a is a variable

δ is a variable

Ω is a constant

A is a constant

How do you find the constants?

Question 2

Equation: $d^3 = \Theta\beta^2 + X$

d is a variable

Θ is a constant

β is a variable

X is a constant

How do you find the constants?

Question 3

Equation: $D = \beta\Delta + y$

D is a constant
β is a variable
Δ is a constant
y is a variable

How do you find the constants?

Question 4

Equation: $\Theta = y \sin(a) + \Lambda$

Θ is a constant
y is a variable
a is a variable
Λ is a constant

How do you find the constants?

Question 5

Equation: $\alpha = \theta \cos(A) + \Delta$

α is a variable

θ is a variable

A is a constant

Δ is a constant

How do you find the constants?

Question 6

Equation: Z = xe^(βY)

Z is a constant

x is a variable

β is a variable

Y is a constant

How do you find the constants?

Question 7

Equation: $\Theta = yD^\lambda$

Θ is a constant

y is a variable

D is a constant

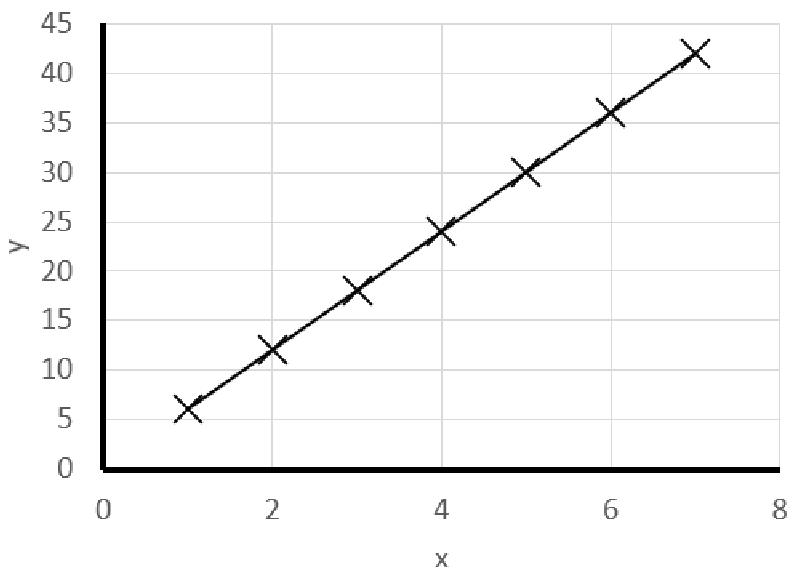
λ is a variable

How do you find the constants?

Question 8

Data:

x	y
1	6
2	12
3	18
4	24
5	30
6	36
7	42

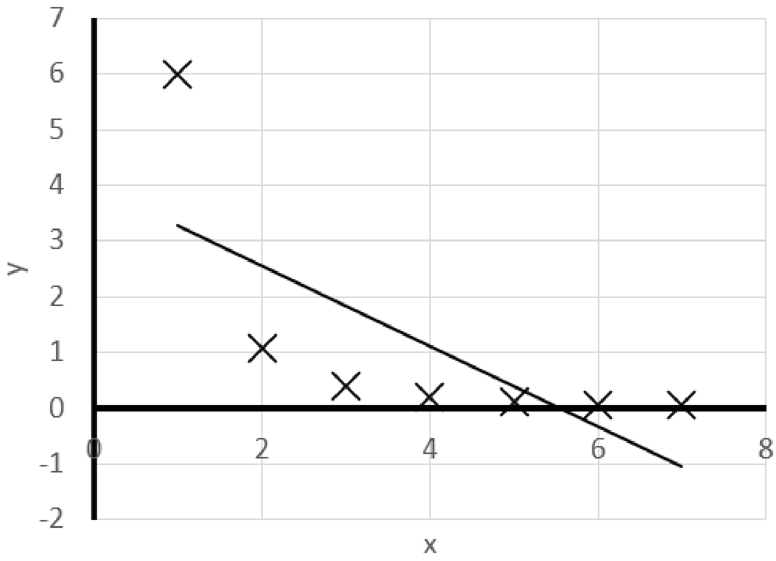


1. What should you plot on the x-axis?
2. What should you plot on the y-axis?
3. In what form will be your equation?
4. What are the constants?

Question 9

Data:

x	y
1	6
2	1.060660172
3	0.384900179
4	0.1875
5	0.107331263
6	0.068041382
7	0.046281364



1. What should you plot on the x-axis?

2. What should you plot on the y-axis?

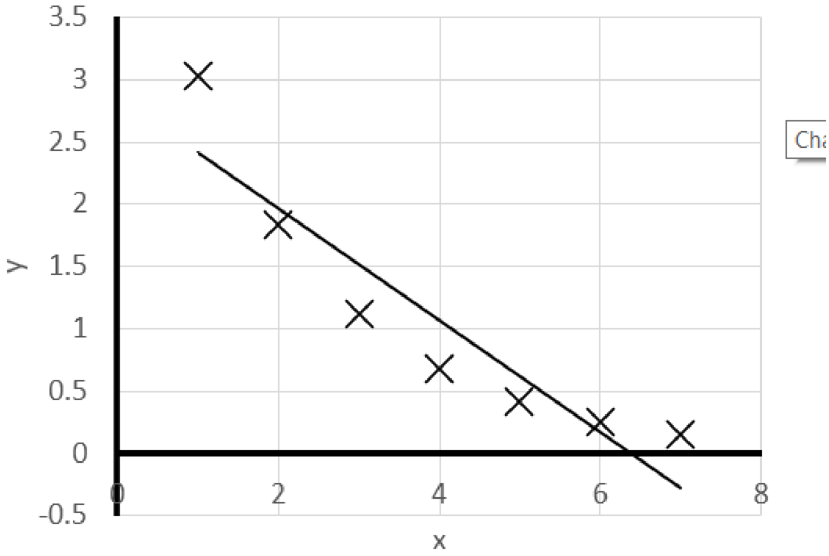
3. In what form will be your equation?

4. What are the constants?

Question 10

Data:

x	y
1	3.032653299
2	1.839397206
3	1.115650801
4	0.676676416
5	0.410424993
6	0.248935342
7	0.150986917

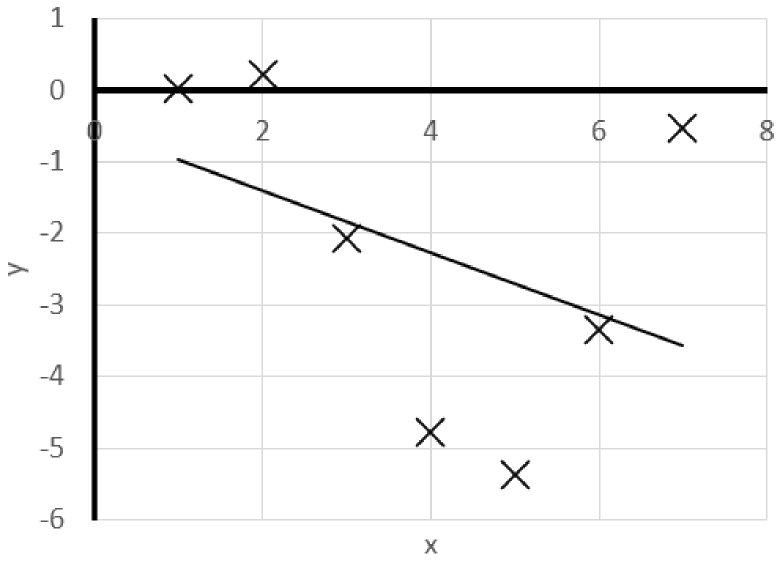


1. What should you plot on the x-axis?
2. What should you plot on the y-axis?
3. In what form will be your equation?
4. What are the constants?

Question 11

Data:

x	y
1	0.024412954
2	0.22789228
3	-2.07663998
4	-4.77040749
5	-5.37677282
6	-3.33824649
7	-0.5290402

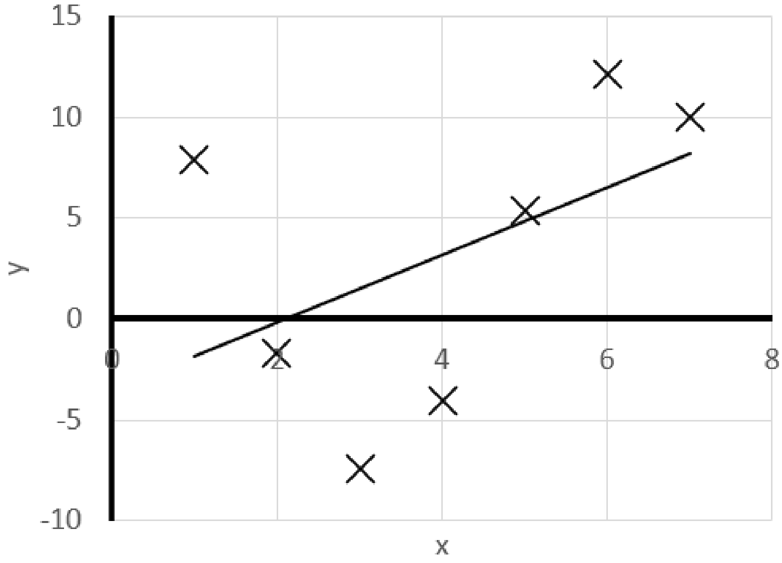


1. What should you plot on the x-axis?
2. What should you plot on the y-axis?
3. In what form will be your equation?
4. What are the constants?

Question 12

Data:

x	y
1	7.903023059
2	-1.66146837
3	-7.39992497
4	-4.03643621
5	5.336621855
6	12.10170287
7	10.03902254



1. What should you plot on the x-axis?
2. What should you plot on the y-axis?
3. In what form will be your equation?
4. What are the constants?